Mark Scheme (Results)

January 2010

GCE

Further Pure Mathematics FP2 (6675)

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Question Number	Scheme	Marks
Q1	(a) $\frac{x^2}{4a^2} - \frac{y^2}{a^2} = 1$	
	Using "$b^2 = a^2(e^2 - 1)$"	
	$a^2 = 4a^2(e^2 - 1)$	M1 A1
	Leading to $e = \frac{\sqrt{5}}{2}$	A1 (3)
	(b) "$x = \frac{a}{e}$" $\Rightarrow$ $x = \frac{2a}{e}$	
	$a = \frac{ex}{2} = \frac{5\sqrt{5}}{2}$ ft their e	M1 A1ft (2)
		[5]
Q2	$\rho = \frac{\mathrm{d}s}{\mathrm{d}\psi} = \tan\psi$	B1
	$s = \int \tan\psi \,\mathrm{d}\psi = \ln\sec\psi \quad (+A)$	M1 A1
	At $s = 0$, $\psi = \frac{\pi}{4}$ $\quad 0 = \ln\sqrt{2} + A \Rightarrow A = -\ln\sqrt{2}$	M1 A1 (5)
	$\left(s = \ln\left(\frac{\sec\psi}{\sqrt{2}}\right)\right)$	[5]

Question Number	Scheme		Marks	
Q3	(a) $2\sinh x\cosh x = 2\left(\frac{e^{x}-e^{-x}}{2}\right)\left(\frac{e^{x}+e^{-x}}{2}\right)$			
	$= \frac{e^{2x}-e^{-2x}}{2}$		M1	
	$= \sinh 2x$ ∗	cso	A1	(2)
	(b) $x = 0$		B1	
	$2\sinh x\cosh x = 6\sinh^{2} x + 7\sinh x$		M1	
	$2\cosh x = 6\sinh x + 7$			
	$e^{x}+e^{-x} = 3e^{x}-3e^{-x}+7$		M1	
	$2e^{2x}+7e^{x}-4=0$		A1	
	$\left(2e^{x}-1\right)\left(e^{x}+4\right)=0$		M1	
	$e^{x} = \frac{1}{2}$		A1	
	$x = -\ln 2$	accept $\ln\frac{1}{2}$	A1	(7)
				[9]

Question Number	Scheme	Marks
Q4	(a) $I_n = \int 1.(\ln x)^n \, \mathrm{d}x$	
	$= x(\ln x)^n - \int x.n(\ln x)^{n-1} \times \frac{1}{x} \mathrm{d}x$	M1 A1 A1
	$= x(\ln x)^n - n\int (\ln x)^{n-1} \, \mathrm{d}x$	
	$= n(\ln x)^n - nI_{n-1}$ * cso	A1 (4)
	(b) $I_3 = x(\ln x)^3 - 3I_2$	M1
	$= x(\ln x)^3 - 3\left(x(\ln x)^2 - 2I_1\right)$	M1
	$= x(\ln x)^3 - 3x(\ln x)^2 + 6I_1$	
	$= x(\ln x)^3 - 3x(\ln x)^2 + 6(x\ln x - I_0)$	
	$= x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x$	M1 A1
	With the limits 1 and e	
	$I_3 = \mathrm{e} - 3\mathrm{e} + 6\mathrm{e} - 6\mathrm{e} + 6 = 6 - 2\mathrm{e}$	M1 A1 (6)
		[10]
	If I_1 is integrated directly	
	(b) $I_3 = x(\ln x)^3 - 3I_2$	M1
	$= x(\ln x)^3 - 3\left(x(\ln x)^2 - 2I_1\right)$	
	$= x(\ln x)^3 - 3x(\ln x)^2 + 6I_1$	M1
	$I_1 = \int 1.\ln x \, \mathrm{d}x = x\ln x - \int x \times \frac{1}{x} \mathrm{d}x = x\ln x - x$ Allow if quoted	M1 A1
	$\left(I_3 = x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x\right)$	
	With the limits 1 and e	
	$I_3 = \mathrm{e} - 3\mathrm{e} + 6\mathrm{e} - 6\mathrm{e} + 6 = 6 - 2\mathrm{e}$	M1 A1 (6)

Question Number	Scheme	Marks
Q5	(a) $\dfrac{dy}{dx} = 3\sinh 3x - 4$	M1 A1
	$3\sinh 3x - 4 = 0$	
	$\sinh 3x = \dfrac{4}{3}$	M1
	$3x = \text{arsinh}\dfrac{4}{3}$	
	$x = \dfrac{1}{3}\text{arsinh}\dfrac{4}{3}$	M1
	$= \dfrac{1}{3}\ln\left[\frac{4}{3} + \sqrt{\left(\left(\frac{4}{3}\right)^2 + 1\right)}\right] = \dfrac{1}{3}\ln 3$	A1 (5)
	(b) $\int(\cosh 3x - 4x)\,dx = \dfrac{\sinh 3x}{3} - 2x^2$	M1 A1
	$A = \left[\dfrac{\sinh 3x}{3} - 2x^2\right]_0^{\frac{1}{3}\ln 3} = \dfrac{\sinh(\ln 3)}{3} - \dfrac{2(\ln 3)^2}{9}$	M1 A1
	$= \dfrac{e^{\ln 3} - e^{-\ln 3}}{6} - \ \ldots = \dfrac{3 - \frac{1}{3}}{6} - \ \ldots$	M1
	$= \dfrac{4}{9} - \dfrac{2(\ln 3)^2}{9}$	
	$= \dfrac{2}{9}\left[2 - (\ln 3)^2\right]$ ∗ cso	A1 (6)
		[11]

Question Number	Scheme	Marks
Q6	(a) $x=\frac{a}{u} \Rightarrow \frac{dx}{du}=-\frac{a}{u^2}$	B1
	$\int \frac{1}{x\sqrt{(a^2-x^2)}}dx=\int \frac{1}{\frac{a}{u}\sqrt{\left(a^2-\frac{a^2}{u^2}\right)}}\left(-\frac{a}{u^2}\right)du$	M1 A1
	$=-\frac{1}{a}\int \frac{1}{\sqrt{(u^2-1)}}du$	M1
	$=-\frac{1}{a}\text{arcosh}\,u \quad (+A)$	M1
	$=-\frac{1}{a}\text{arcosh}\left(\frac{a}{x}\right) \quad (+A)$	A1 (6)
	(b) $\int \frac{1}{x\sqrt{(25-x^2)}}dx=-\frac{1}{5}\text{arcosh}\left(\frac{5}{x}\right)$	M1
	$\left[-\frac{1}{5}\text{arcosh}\left(\frac{5}{x}\right)\right]_3^4=\frac{1}{5}\left(\text{arcosh}\left(\frac{5}{3}\right)-\text{arcosh}\left(\frac{5}{4}\right)\right)$	M1 A1
	$=\frac{1}{5}[\ln 3-\ln 2]$	
	$=\frac{1}{5}\ln\frac{3}{2}$	M1 A1 (5)
		[11]
	Alternative for part (a)	
	$x=\frac{a}{\cosh u} \Rightarrow \frac{dx}{du}=-\frac{\sinh u}{\cosh^2 u}$	B1
	$\int \frac{1}{x\sqrt{(a^2-x^2)}}dx=\int \frac{1}{\frac{a}{\cosh u}\sqrt{\left(a^2-\left(\frac{a}{\cosh u}\right)^2\right)}}\left(-\frac{a\sinh u}{\cosh^2 u}\right)du$	M1 A1
	$=-\frac{1}{a}\int 1\,du=-\frac{1}{a}u \quad (+A)$	M1 M1
	$=-\frac{1}{a}\text{arcosh}\left(\frac{a}{x}\right) \quad (+A)$	A1 (6)

Question Number	Scheme		Marks
Q7	(a) $u = k\arcsin 2x,\ \dfrac{du}{dx} = \dfrac{2k}{\sqrt{(1-4x^2)}}$		
	$\dfrac{dy}{du} = \cos u$		
	$\dfrac{dy}{dx} = \cos u \times \dfrac{2k}{\sqrt{(1-4x^2)}}$		M1 A1
	$\cos u = \sqrt{(1-\sin^2 u)} = \sqrt{(1-y^2)}$		
	$\dfrac{dy}{dx} = \sqrt{(1-y^2)} \times \dfrac{2k}{\sqrt{(1-4x^2)}}$		A1
	$\sqrt{(1-4x^2)}\dfrac{dy}{dx} = 2k\sqrt{(1-y^2)}$		
	$(1-4x^2)\left(\dfrac{dy}{dx}\right)^2 = 4k^2(1-y^2)$ ∗	cso	M1 A1 (5)
	Using $u = \arcsin 2x$ is marked similarly.		
	(b) $-8x\left(\dfrac{dy}{dx}\right)^2 + (1-4x^2)2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2} = -8k^2y\dfrac{dy}{dx}$		M1 A1 A1
	$-4x\dfrac{dy}{dx} + (1-4x^2)\dfrac{d^2y}{dx^2} = -4k^2y$		M1
	$(1-4x^2)\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 4k^2y = 0$ ∗	cso	A1 (5)
			[10]
	Alternative for part (a)		
	$\arcsin y = k\arcsin 2x$		
	$\dfrac{1}{\sqrt{(1-y^2)}}\dfrac{dy}{dx} = \dfrac{2k}{\sqrt{(1-4x^2)}}$		M1 A1 A1
	$\sqrt{(1-4x^2)}\dfrac{dy}{dx} = 2k\sqrt{(1-y^2)}$		
	$(1-4x^2)\left(\dfrac{dy}{dx}\right)^2 = 4k^2(1-y^2)$ ∗	cso	M1 A1 (5)

Question Number	Scheme	Marks
Q8	(a) Eliminating y between $y = mx + c$ and $y^2 = 4ax$	
	$(mx+c)^2 = 4ax$	
	$m^2x^2 + 2(mc - 2a)x + c^2 = 0$	M1 A1
	For a tangent "$b^2 - 4ac = 0$"	
	$4(mc-2a)^2 - 4m^2c^2 = 0$	M1
	Leading to $c = \frac{a}{m}$ ∗ cso	A1 (4)
	(b) $y = mx + \frac{a}{m}$ passes through $(4a, 5a)$	
	$5a = 4ma + \frac{a}{m}$	M1
	$4m^2 - 5m + 1 = 0$	A1
	$(4m-1)(m-1) = 0$	M1
	$m = \frac{1}{4},\ 1$	A1
	$y = \frac{1}{4}x + 4a, \quad y = x + a$ both	A1 (5)
	(c) Obtaining the points of contact between the tangents and the curve	
	e.g. Substituting $m = \frac{1}{4}, c = 4a$ into the quadratic in part (a)	
	$\frac{1}{16}x^2 - 2ax + 16a^2 = \left(\frac{x}{4} - 4a\right)^2 = 0 \Rightarrow x = 16a$	M1
	Point has coordinates $(16a, 8a)$	A1
	Substituting $m = 1, c = a$ into the quadratic in part (a)	
	$x^2 - 2ax + a^2 = (x-a)^2 = 0 \Rightarrow x = a$	
	Point has coordinates $(a, 2a)$	A1
	With all methods, M1 is for finding the x- or y-coordinate of one point of contact. A1 is for any two coordinates correct. The second A1 is for all four coordinates correct.	
	$RQ^2 = (16a-a)^2 + (8a-2a)^2 = 261a^2$	M1
	$RQ = a\sqrt{261} \quad (= 3a\sqrt{29})$	A1 (5)
		[14]

Question Number	Scheme		Marks	
Q8	*Alternatives for* part (a)			
	① Let the point of contact of l and C be $(2at, at^2)$			
	$m = \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{2a}{2at} = \frac{1}{t}$		M1	
	l has the form $y = \frac{1}{t}x + c$		A1	
	$(2at, at^2)$ lies on l $\quad 2at = \frac{1}{t}at^2 + c \Rightarrow c = at$		M1	
	$c = at = \frac{a}{m}$ ∗	cso	A1	(4)
	② Eliminate y between $y = mx + \frac{a}{m}$ and $y^2 = 4ax$			
	$\left(mx + \frac{a}{m}\right)^2 = 4ax$			
	$m^2x^2 + 2ax + \frac{a^2}{m^2} = 4ax$			
	$m^2x^2 - 2ax + \frac{a^2}{m^2} = 0$		M1 A1	
	$\left(mx - \frac{a}{m}\right)^2 = 0$		M1	
	As this is a complete square, l is a tangent to C when $c = \frac{a}{m}$ ∗	cso	A1	(4)
	Note: Alternative ② allows the coordinates of R and Q in part (c) to be written down using $x = \frac{a}{m^2}$.			
	Alternative for part (b)			
	$y = mx + \frac{a}{m} = \frac{x}{t} + at$ passes through $(4a, 5a)$			
	$5a = \frac{4a}{t} + at$		M1	
	$t^2 - 5t + 4 = 0$		A1	
	$(t-1)(t-4) = 0$		M1	
	$t = 1, 4$		A1	
	$y = \frac{1}{4}x + 4a, \quad y = x + a$	both	A1	(5)

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